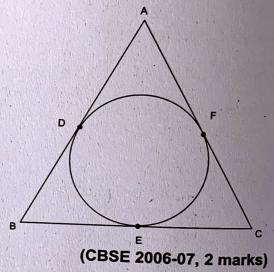


## CIRCLES (TANGENT TO CIRCLES)

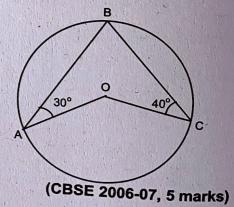
 Prove that the angle subtended by an arc are at the centre is double the angle subtended by it at any point on the remaining part of the circle. Using the above, find x.

C 40° X C (CBSE 2005-06, 6 marks)

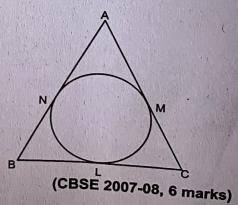
In figure, incircle of  $\triangle ABC$  touches its sides AB, BC and CA at D, E and F respectively. If AB = AC, prove that BE = EC



Prove that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle. Using the above, prove the following: O is the centre of the circle. If  $\angle BAO = 30^{\circ}$  and  $\angle BCO = 40^{\circ}$ , find the value of  $\angle AOC$ .



4. Prove that the lengths of tangents drawn from an external point to a circle are equal. Using the above, prove the following:
ABC is an isosceles triangle in which
AB = AC, circumscribed about a circle, as shown in figure. Prove that the base is bisected by the point of contact.



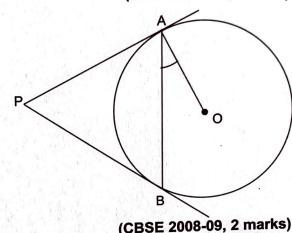


In figure CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm, and BC = 7 cm, then find the length of BR.

R В

(CBSE 2008-09, 1 mark)

Two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that  $\angle APB = 2 \angle OAB$ .



(CBSE 2008-09, 2 marks)

If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then the length of 7. each tangent is:

(A) 
$$\frac{3\sqrt{3}}{2}$$
 cm

(B)  $2\sqrt{3}$  cm

(C) 
$$3\sqrt{3}$$
 cm

(D) 6 cm

From a point A which is at a distance of 10 cm from the centre O of a circle of radius 6 cm, the pair of tangents AB and AC to the circle are drawn. Then the area of the quadrilateral ABOC is: (CBSE 2011-12, 1 mark) 8.

(A) 24 cm<sup>2</sup>

(C) 96 cm<sup>2</sup>

(B) 48 cm<sup>2</sup> (D) 100 cm<sup>2</sup>

To draw a pair of tangent to a circle which are inclined to each other at an angle of 35° it is To draw a pair or large to a short of those two radii of the circle, the angle between them is: 9. (B) 110°

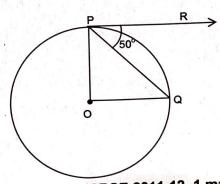
(A) 70°

(D) 145°

(C) 80°

In the figure, if O is centre of a circle, PQ is a chord and the tangent at P makes an angle of 50° with PQ, 10. then ∠POQ is equal to:

- (A) 100°
- (B) 90°
- (C)80°
- (D) 75°



(CBSE 2011-12, 1 mark)



- 11. In two concentric circles, prove that all chords of the outer circle which touches the inner circle are of equal lengths. (CBSE 2011-12, 2 marks)
- 12. In the figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that ∠AOB = 90° (CBSE 2011-12, 3 marks)

 $\begin{array}{c} X \\ \\ \\ \\ \\ X' \end{array}$ 

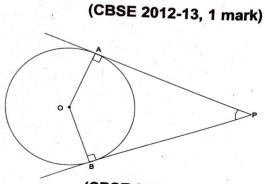
- 13. From a point P, two tangents PA and PB are drawn to a circle C (O, r). If OP = 2r, show that  $\triangle$ APB is equilateral. (CBSE 2011-12, 3 marks)
- 14. Prove that the length of tangents drawn from an external point to a circle are equal. (CBSE 2011-12, 4 marks)
- In figure, a circle with centre O is inscribed in a quadrilateral ABCD such that, it touches the sides BC, AB, AD and CD at points P, Q, R and S respectively. If AB = 29 cm, AD = 23,  $\angle$ B = 90° and DS = 5 cm, then the radius of the circle (in cm.) is
  - (A) 11
  - (B) 6
  - (C) 18
  - (D) 15
- 16. In figure, PA and PB are two tangents drawn from an external point P to a circle with centre O and radius 4 cm. If PA \(\pm\) PB, then the length of each tangent is:
  - (A) 3 cm

(B) 5 cm

S

(C) 4 cm

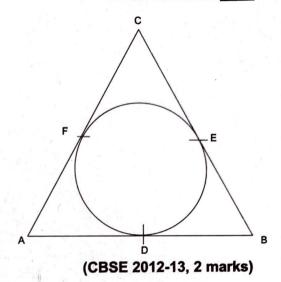
(D) 6 cm



(CBSE 2012-13, 1 mark)



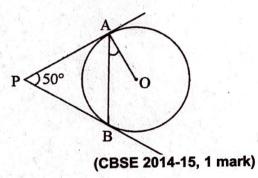
17. In figure, a circle inscribed in triangle ABC touches its sides AB, BC and AC at points D, E and F respectively. If AB = 12 cm, BC = 8 cm and AC = 10 cm, then find the length of AD, BE and CF.



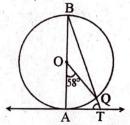
18. Prove that the parallelogram circumscribing a circle is a rhombus

(CBSE 2012-13, 2 marks)

- 19. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. (CBSE 2012-13, 4 marks)
- 20. In figure, PA and PB are tangents to the circle with centre O such that  $\angle APB = 50^{\circ}$ . Write the measure of  $\angle OAB$ .

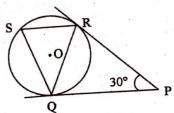


21. In figure, AB is the diameter of a circle with centre O and AT is a tangent. If  $\angle AOQ = 58^{\circ}$ , find  $\angle ATQ$ .



(CBSE 2014-15, 2 marks)

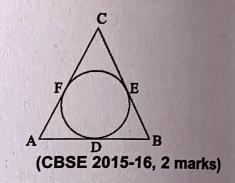
- 22. From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the right bisector of line segment PQ. (CBSE 2014-15, 2 mark)
- 23. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of (CBSE 2014-15, 4 marks) contact.
- 24. In figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that  $\angle$ RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find  $\angle$ RQS.



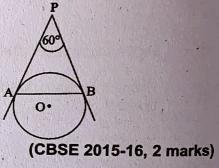
(CBSE 2014-15, 4 marks)



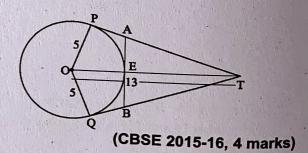
- 25. From an external point P, tangent PA and PB are drawn to a circle with centre O. If  $\angle PAB = 50^{\circ}$ , then find  $\angle AOB$ . (CBSE 2015-16, 1 mark)
- 26. In figure, a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at point D, E and F respectively. If the length of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF.



27. In figure, AP and BP are tangents to a circle with centre O, such that AP = 5 cm and ∠APB = 60°. Find the length of chord AB.

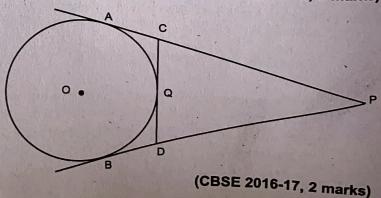


- 28. Prove that the lengths of tangents drawn from an external point to a circle are equal. (CBSE 2015-16, 4 marks)
- 29. In figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



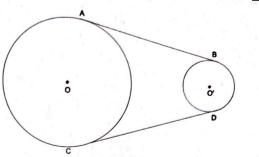
- Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. (CBSE 2015-16, 4 marks)
- 31. Prove that tangents drawn at the ends of a diameter of a circle are parallel to each other.

  (CBSE 2016-17, 2 marks)
- 32. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If PA = 12 cm, QC = QD = 3 cm, then find PC + PD.





33. In the figure, AB and CD are common tangents to two circles of unequal radii. Prove that AB = CD.

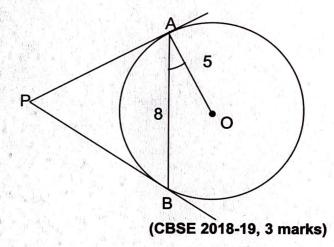


(CBSE 2016-17, 2 marks)

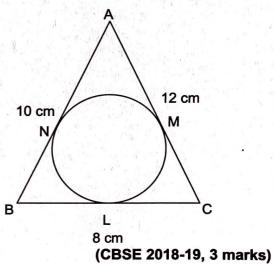
- Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that  $\angle PTQ = 2\angle OPQ$ . (CBSE 2016-17, 3 marks)
- 35. Prove that the lengths of tangents drawn from an external point to a circle are equal.

  (CBSE 2016-17, 4 marks)
- 36. Prove that the lengths of tangents drawn from an external point to a circle are equal.

  (CBSE 2017-18, 3 marks)
- 37. In figure, AB is a chord of length 8 cm of a circle of radius 5 cm and centre O. The tangents at A and B intersect at point P. Find the length of AP.



38. In figure, a circle is inscribed in a ΔABC having side BC = 8 cm, AB = 10 cm and AC = 12 cm. Find the lengths, BL, CM and AN.

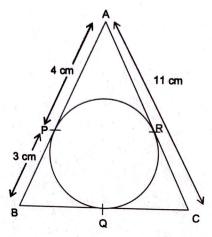


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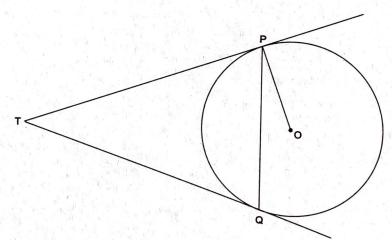


In figure, ΔABC is circumscribing a circle, the length of BC is \_\_\_\_\_ cm.

(CBSE 2019-20, 1 mark)

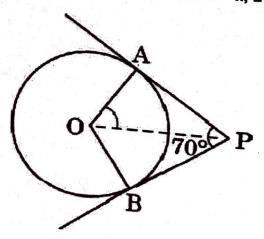


In Figure, two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that  $\angle PTQ = 2\angle OPQ$ . (CBSE 2019-20, 2 marks)



Two concentric circles are of radii 4 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.
 (CBSE 2020-21, Term – II, 2 mark)

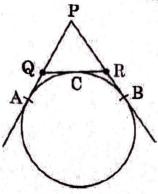
In Figure, if tangents PA and PB drawn from a point P to a circle with centre O, are inclined to each other at an angle of 70°, then find the measure of ∠POA.



(CBSE 2020-21, Term - II, 2 mark)

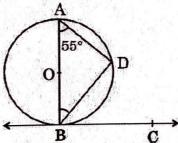


43. (a) In Figure, perimeter of ΔPQR is 20 cm. Find the length of tangent PA.



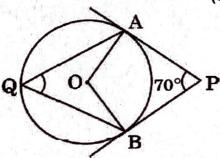
OR

(b) In Figure BC is tangent to the circle at point B of circle centred at O. BD is a chord of the circle so that  $\angle BAD = 55^{\circ}$ . Find m $\angle DBC$ . (CBSE 2020-21, Term – II, 2 mark)



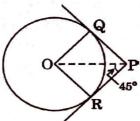
44. If Figure, PA and PB are tangents to the circle with centre at O. If ∠APB = 70°, then find m∠AQB.

(CBSE 2020-21, Term – II, 2 mark)

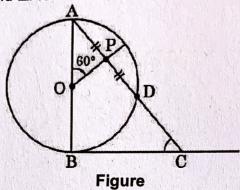


The distance between two tangents parallel to each other of a circle is 13 cm. Find the radius of (CBSE 2021-22 TERM-II, 2 mark) the circle.

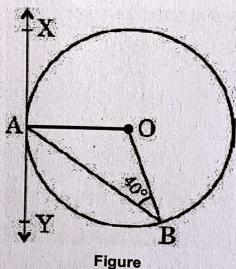
46. In figure, PQ and PR are tangents to the circle centred at O. If ∠OPR = 45°, then prove that (CBSE 2021-22, TERM-II, 2 mark) ORPQ is a square.



47. (a) In figure, AB is diameter of a circle centered at O. BC is tangent to the circle at B. If Op bisects the chord AD and ∠AOP = 60°, then find m∠C.

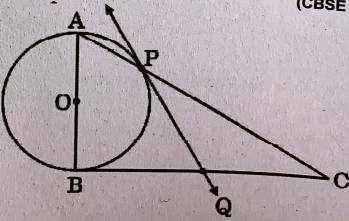


(b) In figure, XAY is a tangent to the circle centered at O. If ∠ABO = 40°, then find m∠BAY and m∠AOB.
(CBSE 2021-22 TERM-II, 2 mark)



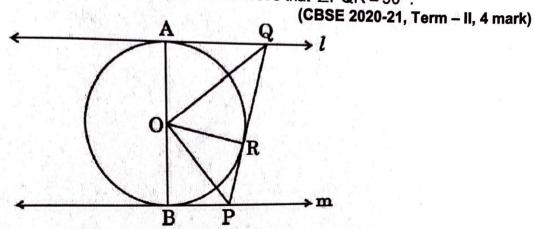
In Figure, a triangle ABC with ∠B = 90° is shown. Taking AB as diameter, a circle has been drawn intersecting AC at point P. Prove that the tangent drawn at point P bisects BC.

(CBSE 2020-21, Term – II, 4 mark)





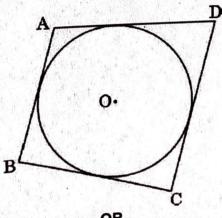
In Figure, the tangent  $\ell$  is parallel to the tangent m drawn at points A and B respectively to a 49. circle centred at O. PQ is a tangent to the circle at R. Prove that  $\angle PQR = 90^{\circ}$ .



(a) Prove that a parallelogram circumscribing a circle is a rhombus. 50.

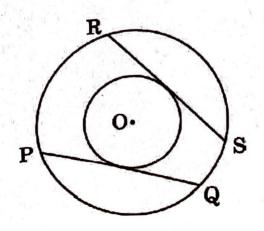
(b) Prove that the perpendicular at the point of contact to the tangent to a circle passes through (CBSE 2020-21, Term - II, 4 mark) the centre of the circle.

(a) In Figure, quadrilateral ABCD circumscribes a circle centred at O. Prove that AD + BC = AB + 51. CD



(b) In Figure, two concentric circles are drawn with centre O. PQ and RS are two chords of the

In Figure, two concentro of the smaller circle. Prove that PQ = RS. larger circle which are tangents to the smaller circle. (CBSE 2020-21, Term - II, 4 mark)

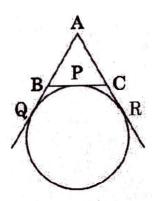




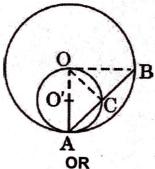
- 52. (a) Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

  OR
  - (b) If a circle is touching the side BC of  $\triangle$ ABC at P and is touching AB and AC produced at Q and R respectively (see the figure). Prove that  $AQ = \frac{1}{2}$  (perimeter of  $\triangle$ ABC).

(CBSE 2021-22 TERM-II, 4 mark)

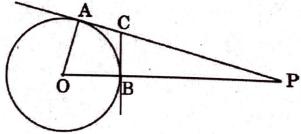


53. (a) In figure, two circles with centres at O and O' of radii 2r and r respectively, touch each other internally at A. A chord AB of the bigger circle meets the smaller circle at C. Show that C bisects AB.

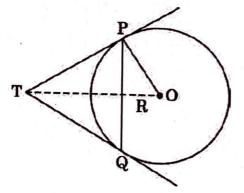


(b) In figure, O is centre of a circle of radius 5 cm. PA and BC are tangents to the circle at A and B respectively. If OP = 13 cm, then find the length of tangents PA and BC.

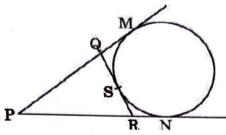
(CBSE 2021-22, TERM-II, 4 mark)



54. In the given figure, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q meet at a point T. Find the length of TP. (CBSE 2021-22 TERM-II, 4 mark)



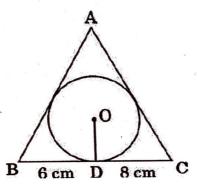
 (a) In figure, if a circle touches the side QR of ΔPQR at S and extended sides PQ and PR at M and N, respectively, then



Prove that  $PM = \frac{1}{2}(PQ + QR + PR)$ 

OR

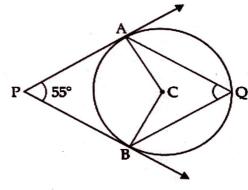
(b) In figure, a triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 6 cm and 8 cm respectively. If the area of ΔABC is 84 cm², find the lengths of sides AB and AC.
(CBSE 2021-22, TERM-II, 4 mark)



56. In the given figure, PA and PB are tangents from external point P to a circle with centre C and Q is any point on the circle. Then the measure of ∠AQB is

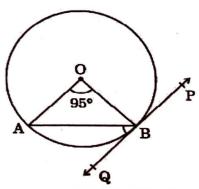


<sup>(</sup>B) 125°



(CBSE 2022-23, 1 mark)

- 57. In the given figure, PQ is tangent to the circle centred at O. If ∠AOB = 95°, then the measure of ∠ABQ will be
  - (A) 47.5°
  - (B) 42.5°
  - (C) 85°
  - (D) 95°



(CBSE 2022-23, 1 mark)

<sup>(</sup>C) 55°

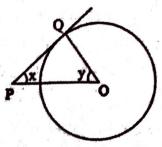
<sup>(</sup>D) 90°

31.



In the given figure, PQ is a tangent to the circle with centre O. If  $\angle OPQ = x$ ,  $\angle POQ = y$ , then x + y is:

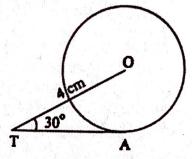
- (A) 45°
- (B) 90°
- (C) 60°
- (D) 180°



(CBSE 2022-23, 1 mark)

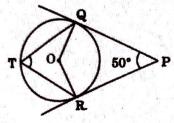
In the given figure, TA is a tangent to the circle with centre O such that OT = 4 cm,  $\angle OTA = 30^{\circ}$ , then length of TA is:

- (A)  $2\sqrt{3}$  cm
- (B) 2 cm
- (C)  $2\sqrt{2}$  cm
- (D)  $\sqrt{3}$  cm



(CBSE 2022-23, 1 mark)

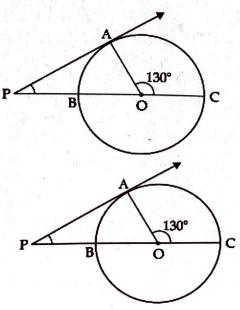
From a point p, two tangents PQ and PR are drawn to a circle with centre at O. T is a point on the major arc QR of the circle. If  $\angle QPR = 50^\circ$ , then  $\angle QTR$  equals:



- (A) 50°
- (C) 65°

- (B) 130° (D) 90°
- (CBSE 2022-23, 1 mark)

In the given figure, PA is a tangent to the circle drawn from the external point P and PBC is the secant to the circle with BC as diameter. If  $\angle AOC = 130^{\circ}$ , then find the measure of  $\angle APB$ , where O is the centre of the circle.



(CBSE 2022-23, 2 marks)

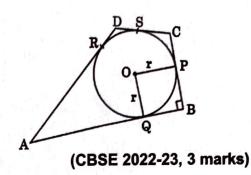


66.

68.

The centre of circle is (2a, a - 7). Find the values of 'a' if the circle passes through the point (11, -9). Radius of the circle is  $5\sqrt{2}$  cm. (CBSE 2022-23, 3 marks)

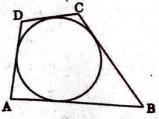
In the given figure, a circle is inscribed in a quadrilateral ABCD in which  $\angle B = 90^{\circ}$ . If AD = 17 cm, AB = 20 cm and DS = 3 cm, then find the radius of the circle.



- Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

  (CBSE 2022-23, 3 marks)
- Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle. (CBSE 2022-23, 3 marks)

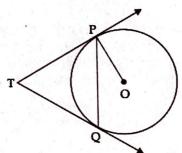
A quadrilateral **ABCD** is drawn to circumscribe a circle, as shown in the figure. Prove that AB + CD = AD + BC



(CBSE 2022-23, 3 marks)

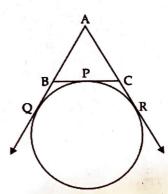
(CBSE 2022-23, 3 marks)

- 67. Prove that the parallelogram cicmscribing a circle is a rhombus.
  - Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that  $\angle PTQ = 2\angle OPQ$ .



(CBSE 2022-23, 5 marks)

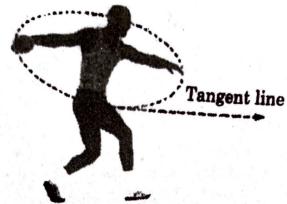
69. A circle touches the side BC of a  $\triangle$ ABC at a point P and touches AB and AC when produced at Q and R respectively. Show that AQ =  $\frac{1}{2}$  (Perimeter of  $\triangle$ ABC).



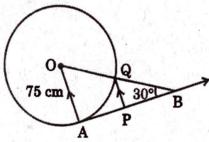
(CBSE 2022-23, 5 marks)

## 214

The discus throw is an event in which an athlete attempts to throw a discus. The athlete spins The discus throw is an event in which an author attempts to the throw is an event in which an author attempts to the throw is an event in which an author activities are the throw. When anti-clockwise around one and a half times through a circle, then releases the throw. When 70. released, the discus travels along tangent to the circular spin orbit.



In given figure, AB is one such tangent to circle of radius 75 cm. Point O is centre of the circle and ∠ABO = 30°. PQ is parallel to OA.



Based on above information:

- (a) Find the length of AB.
- (b) Find the length of OB.
- (c) Find the length of AP.

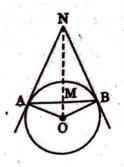
OR

Find the length of PQ.

1 mark 1 mark 2 marks

(CBSE 2022-23, 4 marks)

Circles play an important part in our life. When a circular 71. object is hung on the wall with a cord at nail N, the cords NA and NB work like tangents. Observe the figure, given that  $\angle ANO = 30^{\circ}$  and OA = 5 cm.



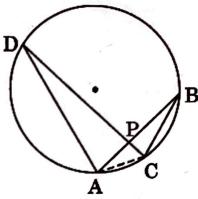
Based on the above, answer the following questions:

- (i) Find the distance AN.
- (ii) Find the measure of ∠AOB.
- (iii) (a) Find the total length of cords NA, NB and the chord AB.
- (iii) (b) If ∠ANO is 45°, then name the type of quadrilateral OANB. Justify your answer.

(CBSE 2022-23, 4 marks)

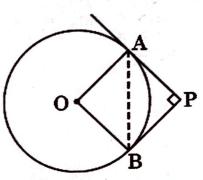


- 72. AB and CD are two chords of a circle intersecting at P. Choose the correct statement from the following:
  - (A)  $\triangle$ ADP ~  $\triangle$ CBA
  - (B)  $\triangle$ ADP  $\sim$   $\triangle$ BPC
  - (C)  $\triangle$ ADP ~  $\triangle$ BCP
  - (D)  $\triangle$ ADP  $\sim$   $\triangle$ CBP



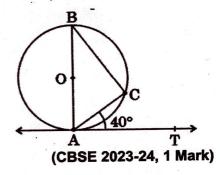
(CBSE 2023-24, 1 Mark)

- 73. In the given figure, tangents PA and PB to the circle centered at O, from point P are perpendicular to each other. If PA = 5 cm, then length of AB is equal to:
  - (A) 5 cm
  - (B)  $5\sqrt{2}$  cm
  - (C)  $2\sqrt{5}$  cm
  - (D) 10 cm



(CBSE 2023-24, 1 Mark)

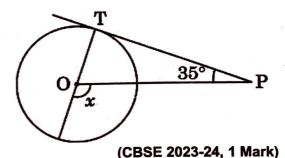
- 74. In the given figure, AT is tangent to a circle centered at O. If ∠CAT = 40°, then ∠CBA is equal to:
  - (A) 70°
  - (B) 50°
  - (C) 65°
  - (D) 40°



- 75. Maximum number of common tangents that can be drawn to two circles intersecting at two (CBSE 2023-24, 1 Mark)
  - distinct point is:
  - (A) 4

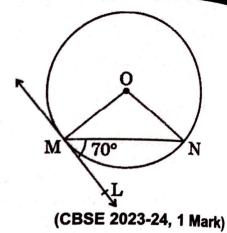
(C) 2

- (B) 3
- (D) 1
- 76. In the given figure, if PT is a tangent to a circle with centre O and  $\angle$ TPO = 35°, then the measure of  $\angle$ x is

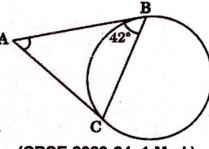


- (A) 110°
- (B) 115°
- (C) 120°
- (D) 125°

- 77. In the given figure, O is the centre of the circle. MN is the chord and the tangent ML at point M makes an angle of 70° with MN. The measure of ∠MON is:
  - (A) 120°
  - (B) 140°
  - (C) 70°
  - (D) 90°

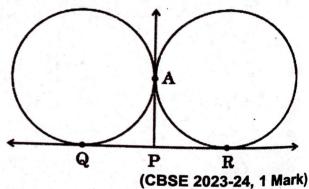


- 78. In the given figure, AB and AC are tangents to the circle. If  $\angle$ ABC = 42°, then the measure of  $\angle$ BAC is
  - (A) 96°
  - (B) 42°
  - (C) 106°
  - (D) 86°

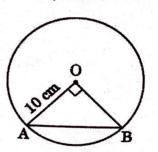


(CBSE 2023-24, 1 Mark)

- 79. In the given figure, QR is a common tangent to the two given circles touching externally at A. The tangent at A meets QR at P. If AP = 4.2 cm, then the length of QR is:
  - (A) 4.2 cm
  - (B) 2.1 cm
  - (C) 8.4 cm
  - (D) 6.3 cm

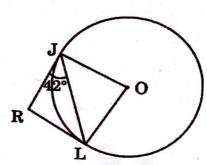


- 80. A chord of a circle of radius 10 cm subtends a right angle at its centre. The length of the chord (in cm) is:
  - (A) 5√2
  - (B)  $10\sqrt{2}$
  - (C)  $\frac{5}{\sqrt{2}}$
  - (D) 5



(CBSE 2023-24, 1 Mark)

- 81. In the given figure, RJ and RL are two tangents to the circle. If  $\angle$ RJL = 42°, then the measure of  $\angle$ JOL is
  - (A) 42°
  - (B) 84°
  - (C) 96°
  - (D) 138°



(CBSE 2023-24, 1 Mark)

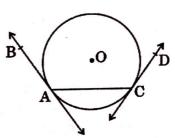


pirection: In Q. No 82 is a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.

- (A) both, Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).
- (B) both, Assertion (A) and Reason (R) are true and Reason (R) is not correct explanation of Assertion (A).
- (C) Assertion (A) is true but Reason (R) is false.
- (D) Assertion (A) is false but Reason (R) is true.
- Assertion (A): The tangents drawn at the end points of a diameter of a circle are parallel.

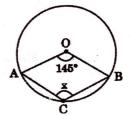
  (CBSE 2023-24, 1 Mark)

  Reason (R): Diameter of a circle is the longest chord.
- 83. (a) If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then find the length of each tangent. (CBSE 2023-24, 2 Marks)
  - (b) Prove that the tangents drawn at the ends of a diameter of a circle are parallel.
- 84. In the given figure, AB and CD are tangents to a circle centered O. Is ∠BAC = ∠DCA? Justify your answer.



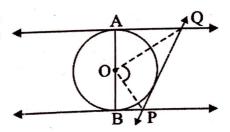
(CBSE 2023-24, 2 Marks)

85. In the given figure, O is the centre of the circle. If  $\angle AOB = 145^{\circ}$ , then find the value of x.



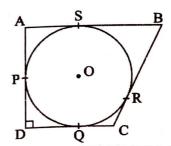
(CBSE 2024, 2 Marks)

86. (a) In the given figure, AB is a diameter of the circle with centre O. AQ, BP and PQ are tangents to the circle. Prove that ∠POQ = 90°.



OR

(b) A circle with centre O and radius 8 cm is inscribed in a quadrilateral ABCD in which P, Q, R, S are the points of contact as shown. If AD is perpendicular to DC, BC = 30 cm and BS = 24 cm, then find the length DC.



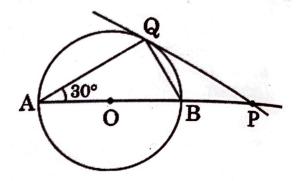
(CBSE 2023-24, 3 Marks)



Prove that the parallelogram circumscribing a circle is a rhombus.

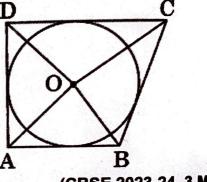
(CBSE 2023-24, 3 Marks)

88. (a) In the given figure, PQ is tangent to a circle centered at O and ∠BAQ = 30°; show that BP = BQ



OR

(b) In the given figure, AB, BC, CD and DA are tangents to the circle with centre O forming a quadrilateral ABCD. Show that ∠AOB + ∠COD = 180°

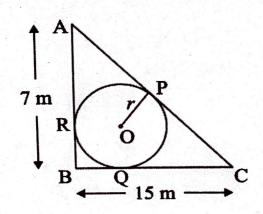


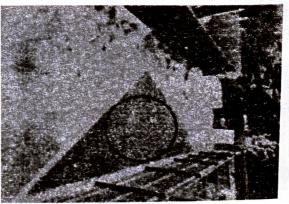
(CBSE 2023-24, 3 Marks)

89. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord. (CBSE 2023-24, 3 Marks)

## **Case Study**

90. A backyard is in the shape of a triangle ABC with right angle at B. AB = 7 m and BC = 15 m. A circular pit was dug inside it such that it touches the walls AC, BC and AB at P, Q and R respectively such that AP = x m. (CBSE 2023-24, 4 Marks)





Based on the above information, answer the following questions:

- (i) Find the length of AR in terms of x.(ii) Write the type of quadrilateral BQOR.
- (iii) (a) Find the length PC in terms of x and hence find the value of x.
  - (b) Find x and hence find the radius r of circle.